

Blandade övningar Kap 1 M 3b

$$\textcircled{1} \quad (3x+2)^2 - (2x-3)^2 = 9x^2 + 12x + 4 - (4x^2 - 12x + 9) = \\ = 9x^2 + 12x + 4 - 4x^2 + 12x - 9 = 5x^2 + 24x - 5$$

$$\textcircled{2} \quad \frac{10-2x}{5-x} = \frac{2(5-x)}{(5-x)} = 2$$

Wilma $\frac{64+x}{4} = 16+x$ Fel

Jool $\frac{64+x}{4} = 16 + \frac{x}{4}$ Rätt både 64 och x måste delas med 4.

$$\textcircled{4} \quad 4^{\frac{1}{2}} + 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = \sqrt{4} + 5^{\frac{1}{2} + \frac{1}{2}} = 2 + 5^1 = 7$$

$$\textcircled{5} \quad \frac{x-2}{2x^2-8x} = \frac{x-2}{2x(x-4)} \quad x=0 \text{ och } x=4$$

$$\textcircled{6} \quad \frac{5+4}{5^2-16} = \frac{(5+4)}{(5-4)(5+4)} = \frac{1}{5-4}$$

$$\textcircled{7} \quad (x+a)^2 - (x-a)^2 = x^2 + 2xa + a^2 - (x^2 - 2xa + a^2) = \\ = \cancel{x^2} + 2xa + \cancel{a^2} - \cancel{x^2} + 2xa - \cancel{a^2} = 4xa$$

$$\text{b) } x(x+2)^2 - x^3 = x(x^2 + 4x + 4) - x^3 = x^3 + 4x^2 + 4x - x^3 = \\ = 4x^2 + 4x$$

$$\textcircled{8} \text{ a) } (x-1)(x+1) = 0$$

$$x_1 = 1 \quad x_2 = -1$$

$$\text{b) } \frac{5 \cdot 10^x}{5} = \frac{10}{5}$$

$$10^x = 2$$

$$\lg 10^x = \lg 2$$

$$x \lg 10 = \lg 2$$

$$x = \lg 2$$

$$\text{c) } 2x + 4 = \frac{6}{x} \quad \text{Multipl. mit } x$$

$$\frac{2x^2 + 4x}{2} = \frac{6}{2}$$

$$x^2 + 2x = 3$$

$$x^2 + 2x - 3 = 0$$

$$x = -1 \pm \sqrt{1^2 + 3}$$

$$x = -1 \pm \sqrt{4}$$

$$x_1 = -1 + 2 = 1$$

$$x_2 = -1 - 2 = -3$$

$$\text{d) } \frac{2x^5}{2} = \frac{6}{2}$$

$$x^5 = 3$$

$$x = \sqrt[5]{3}$$

$\textcircled{9}$ Sc. part.

$$\textcircled{10} \text{ a) } \frac{5}{x+2} - \frac{3-x}{x+2} = \frac{5 - (3-x)}{x+2} = \frac{5-3+x}{x+2} = \frac{2+x}{x+2} = \frac{x+2}{x+2} = 1$$

$$\text{b) } \frac{2}{x-2} - \frac{5}{2-x} = \frac{2}{x-2} - \frac{5}{-1(x-2)} = \frac{2}{x-2} + \frac{5}{x-2} = \frac{2+5}{x-2} = \frac{7}{x-2}$$

$$(11) \quad x^2 - 10x + a = 0 \quad x=3 \quad x=7$$

$$3^2 - 10 \cdot 3 + a = 0$$

$$9 - 30 + a = 0$$

$$-21 + a = 0$$

$$a = 21$$

(12)

$$\frac{(2a^{-2})^3}{2a^2 + 2a^2} = \frac{2^3 \cdot a^{-6}}{4a^2} = \frac{8a^{-6}}{4a^2} = 2a^{-6-2} = 2a^{-8}$$

(13)

$$5x^4 - 8x - 3x^4 + 6x = 0$$

$$\frac{2x^4}{2} - \frac{2x}{2} = 0$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$\begin{array}{l} / \quad \backslash \\ x_1 = 0 \quad x^3 - 1 = 0 \end{array}$$

$$x^3 = 1$$

$$x = \sqrt[3]{1}$$

$$x = 1$$

Skärningspunkter

(14)

$$f(1) = 4 \cdot 1 - 1^2 - 3 = 4 - 1 - 3 = 0$$

$$f(4) = 4 \cdot 4 - 4^2 - 3 = 16 - 16 - 3 = -3$$

$$(1, 0)$$

$$(4, -3)$$

$$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{4 - 1} = \frac{-3}{3} = -1$$

$$y = kx + m$$

$$k = -1$$

$$x = 1$$

$$y = 0$$

$$0 = -1 \cdot 1 + m$$

$$m = 1$$

$$\text{Svar: } y = -x + 1$$

$$(15) \quad a) \quad \frac{30000 + 15x}{x}$$

$$b) \quad \frac{30000 + 15x}{x} = 40$$

$$30000 + 15x = 40x$$
$$-15x \quad -15x$$

$$30000 = 25x$$

$$x = \frac{30000}{25}$$

$$x = 1200 \text{ mil}$$

(16)

$$f(-1) = 2 \Rightarrow \begin{cases} -2 = a \cdot (-1)^2 + b \cdot (-1) & (1) \end{cases}$$

$$f(1) = 6 \Rightarrow \begin{cases} 6 = a \cdot (1)^2 + b \cdot (1) & (2) \end{cases}$$

$$+ \begin{cases} -2 = a - b & (1) \\ 6 = a + b & (2) \end{cases}$$

$$\frac{4}{2} = \frac{2a}{2}$$

$$a = 2 \text{ insert in (1)}$$

$$-2 = 2 - b$$

$$-4 = -b$$

$$b = 4$$

$$\text{SVWV: } a = 2 \quad b = 4$$

Fel i uppgiften +5 istället för -5

$$(17) \quad x^3 - x(6x+5) = 0$$

$$x^3 - 6x^2 + 5x = 0$$

$$x(x^2 - 6x + 5) = 0$$

$$x_1 = 0$$

$$x^2 - 6x + 5 = 0$$

$$x = 3 \pm \sqrt{9+5}$$

$$x = 3 \pm \sqrt{4}$$

$$x = 3 \pm 2$$

$$x_2 = 5$$

$$x_3 = 1$$

$$\text{Svar: } x_1 = 0 \quad x_2 = 5 \quad x_3 = 1$$

$$(18) \quad a) 1 = \sqrt{3 \cdot 15} = \sqrt{3 \cdot 15} = \sqrt{45}$$

$$\sqrt{36} = 6 \quad \sqrt{49} = 7 \quad \Rightarrow \quad 6 < a < 7 \quad \text{dvs } D$$

$$(19) \quad \frac{5}{4x} + \frac{1}{x} - x = 0 \quad \text{Mult med } 4x$$

$$\frac{4x \cdot 5}{4x} + \frac{4x \cdot 1}{1x} - 4x \cdot x = 0$$

$$5 + 4 - 4x^2 = 0$$

$$9 - 4x^2 = 0$$

$$-4x^2 = -9$$

$$x^2 = \frac{-9}{-4}$$

$$x^2 = 2,25$$

$$x = \pm 1,5$$

$$b) \frac{5}{4x} + \frac{1}{x} - x =$$

Gör mgn: $4x$

$$= \frac{5}{4x} + \frac{1 \cdot 4}{x \cdot 4} - \frac{4x \cdot x}{4x} =$$

$$= \frac{5}{4x} + \frac{4}{4x} - \frac{4x^2}{4x} =$$

$$= \frac{5+4-4x^2}{4x} = \frac{9-4x^2}{4x}$$

(90)

Rationellt uttryck \Rightarrow bråk.

$x \neq 1 \Rightarrow$ nämnaren skall vara $x-1$

Om $f(-2) = 1$ ger $\frac{?}{x-1} = 1$

$$\frac{?}{-2-1} = 1$$

$$\frac{2}{-3} = 1$$

$$? = -3$$

exempel $2 \cdot (-2) + 1 = -3$ ger

$$2 \cdot x + 1$$

Svar: $\frac{2x+1}{x-1}$

(21)

a) nej

b) $x=1$ o $x=5$

c) $f(x) = k(x-a)(x-b)$

$$a = 1$$

$$b = 5$$

$$f(x) = 4$$

$$x = 0$$

$$4 = k \cdot (1-1)(0-5)$$

$$\Rightarrow 4 = k \cdot (-1)(-5)$$

$$4 = k \cdot 5$$

$$k = 0,8$$

svr: $f(x) = 0,8(x-1)(x-5)$

Sätt in 11

$$f(11) = 0,8(11-1)(11-5) = 0,8 \cdot 9 \cdot 6 = 43,2$$

Sätt in 0

$$f(0) = 0,8(0-1)(0-5) = 0,8 \cdot (-1)(-5) = 4$$

$$6 \cdot f(0) = 6 \cdot 4 = 24$$

Nej inte sant

$$f(11) \neq 6 \cdot f(0)$$

$$43,2 \neq 24$$

$$VL \neq HL$$

utan miniräknet

$$(22) a) x^2 + 10x + 25 = (x+5)^2$$

$$\Rightarrow (3495+5)^2 = (4000)^2 = 16\,000\,000$$

$$b) \frac{2x^3 - 50x}{2x^2 - 10x} = \frac{2x(x^2 - 25)}{2x(x-5)} = \frac{(x-5)(x+5)}{(x-5)} =$$

$$= x+5$$

$$\text{om } 3495+5 = 4000$$

(23)

$$\frac{x+1}{x} - \frac{x}{x+1} = \frac{3}{2}$$

Multiplikera med $2x(x+1)$

$$\frac{2x(x+1)(x+1)}{x} - \frac{2x(x+1) \cdot x}{x+1} = \frac{2x(x+1) \cdot 3}{2}$$

$$2(x+1)(x+1) - 2x^2 = 3x(x+1)$$

$$2(x^2 + 2x + 1) - 2x^2 = 3x^2 + 3x$$

$$2x + 2 = 3x^2 + 3x$$
$$-3x^2 - 3x \quad -3x - 3x$$

$$\frac{-3x^2 - x + 1}{-3} = \frac{0}{-3}$$

$$x^2 + \frac{1}{3}x - \frac{1}{3} = 0$$

pq-formeln ger

$$x = \frac{1}{6} \pm \sqrt{\frac{1}{36} + \frac{24}{36}} \quad \left(\frac{24}{36} = \frac{24}{36} \right)$$

$$x = \frac{1}{6} \pm \sqrt{\frac{25}{36}}$$

$$x_1 = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1$$

$$x_2 = \frac{1}{6} - \frac{5}{6} = -\frac{4}{6} = -\frac{2}{3}$$

$$23 \quad b) \quad \frac{x+1}{x} - \frac{x}{x+1} = \quad \text{Gör MGV } x(x+1)$$

$$= \frac{(x+1)(x+1)}{x(x+1)} - \frac{x \cdot x}{x(x+1)} =$$

$$= \frac{x^2 + 2x + 1 - x^2}{x(x+1)} = \frac{2x+1}{x(x+1)}$$

$$(24) \quad f(x) = 5x^2$$

$$a) \quad \frac{5(2+h)^2}{h} - 5 \cdot 2^2 = \frac{5(4+4h+h^2)}{h} - 5 \cdot 4 =$$

$$= \frac{20 + 20h + h^2 - 20}{h} = \frac{5h^2 + 20h}{h} = \frac{h(5h+20)}{h} =$$

$$= 5h + 20$$

$$b) \quad \frac{5(x+h)^2}{h} - 5 \cdot (x)^2 = \frac{5(x^2 + 2xh + h^2)}{h} - 5x^2 =$$

$$= \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h} = \frac{5h^2 + 10xh}{h} = \frac{h(5h+10x)}{h}$$

$$= 5h + 10x$$

$$(25) \quad x^4 - x^2 = 2$$

ersätt $x^2 = t$ och sätt in

$$t^2 - t = 2$$

$$t^2 - t - 2 = 0$$

pq-formeln ger

$$t = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}}$$

$$t = \frac{1}{2} \pm \sqrt{\frac{9}{4}}$$

$$t = \frac{1}{2} \pm \frac{3}{2}$$

$$t_1 = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

$$t_2 = \frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$$

$$\text{Om } x^2 = t \Rightarrow$$

$$x^2 = 2$$

$$\text{om } t = 2$$

$$x = \pm \sqrt{2} \quad \text{Ja en reell rot}$$

$$\text{om } x^2 = t \Rightarrow$$

$$x^2 = \sqrt{-1} \quad \text{om en icke-reell rot}$$

$$\text{om } t = -1$$

(26) $\frac{1}{x} + \frac{1}{y}$ om $x+y=4$ om $x \cdot y=1$

Förändla $\frac{1}{x} + \frac{1}{y}$ Gör: MAN = $x \cdot y$

$$= \frac{1 \cdot y}{x \cdot y} + \frac{1 \cdot x}{y \cdot x} = \frac{y}{x \cdot y} + \frac{x}{x \cdot y} = \frac{y+x}{x \cdot y} = \frac{4}{1} = 4$$

(27) $ax^3 + bx^2 + cx + d$

d är skärning med y -axeln ty $x=0$ ger $y=d$

Använd sedan faktorsatsen för nollställena

$(-2, 0)$ $(3, 0)$ $(8, 0)$

$$y = k \cdot (x+2)(x-3)(x-8)$$

$$y = k \cdot (x+2)(x^2 - 3x - 8x + 24)$$

$$y = k(x+2)(x^2 - 11x + 24)$$

$$y = k(x^3 - 11x^2 + 24x + 2x^2 - 22x + 48)$$

$$y = k(x^3 - 9x^2 + 2x + 48)$$

Låt $x=0$ om $y=24$ om sätt in

$$24 = k(0^3 - 9 \cdot 0^2 + 48)$$

$$24 = k \cdot 48$$

$$k = 0,5 \quad \text{sätt in } k$$

$$y = 0,5(x^3 - 9x^2 + 2x + 48)$$

$$y = 0,5x^3 - 4,5x^2 + x + 24$$

$$a = 0,5$$

$$b = -4,5$$

$$c = 1$$

$$d = 24$$

(28) a)

$$9 \cdot 3^{2x+1} = 1$$

$$9 \cdot 3^{2x} \cdot 3^1 = 1$$

$$\frac{27 \cdot 3^{2x}}{27} = \frac{1}{27}$$

$$3^{2x} = \frac{1}{27}$$

$$\lg 3^{2x} = \lg\left(\frac{1}{27}\right)$$

$$\frac{2x \cdot \lg 3}{\lg 3} = \frac{\lg\left(\frac{1}{27}\right)}{\lg 3}$$

$$2x = \frac{\lg\left(\frac{1}{27}\right)}{\lg 3}$$

$$x = \frac{\lg\left(\frac{1}{27}\right)}{\frac{\lg 3}{2}}$$

$$x = \frac{\lg\left(\frac{1}{27}\right)}{2 \cdot \lg 3}$$

$$x = \frac{\lg \frac{1}{27}}{\lg 3^2}$$

$$x = \frac{\lg \frac{1}{27}}{\lg 9}$$

$$x = -1,5$$

Alternativ Lösung

$$9 \cdot 3^{2x+1} = 1$$

$$3^2 \cdot 3^{2x+1} = 3^0$$

$$3^{2+2x+1} = 3^0$$

$$3^{3+2x} = 3^0$$

Jvf exponenten

$$3+2x=0$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$x = -1,5$$

$$b) x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$$

Ersatz mit $x^{\frac{1}{3}} = a$, ger

$$\left(x^{\frac{1}{3}}\right)^2 - 5\left(x^{\frac{1}{3}}\right) + 6 = 0$$

$$a^2 - 5a + 6 = 0$$

pg-formeln.

$$a = 2,5 \pm \sqrt{6,25 - 6}$$

$$a = 2,5 \pm \sqrt{0,25}$$

$$a = 2,5 \pm 0,5$$

$$a_1 = 2,5 + 0,5 = 3$$

$$a_2 = 2,5 - 0,5 = 2$$

$$\text{Lat } x^{\frac{1}{3}} = 3 \Rightarrow x = 3^3 = 27$$

$$x^{\frac{1}{3}} = 2 \Rightarrow x = 2^3 = 8$$

$$(29) \quad K(x) = \frac{x^2}{5x+30}$$

$$K(18) = \frac{18^2}{5 \cdot 18 + 30} = \frac{324}{120} = 2,7$$

$$K(14) = \frac{14^2}{5 \cdot 14 + 30} = \frac{196}{100} = 1,96$$

$$K(18) - K(14) = 2,7 - 1,96 = 0,74$$

$$(30) \quad a) \frac{100(3x+2)}{100} = 5 \cdot 100$$

$$3x+2 = 500$$

$$\frac{3x}{3} = \frac{498}{3}$$

$$x = 166$$

$$b) \frac{100 \cdot 3x^2}{100} = 5 \cdot 100$$

$$3x^2 = 500$$

$$x^2 = \frac{500}{3}$$

$$x = \sqrt{\frac{500}{3}}$$

$$c) \frac{100 \cdot 2x^3}{100} = 5 \cdot 100$$

$$\frac{2x^3}{2} = \frac{500}{2}$$

$$x^3 = 250$$

$$x = 6,2999 \approx 6,30$$

$$d) \frac{100 \cdot 3 \cdot 2^x}{100} = 5 \cdot 100$$

$$\frac{3 \cdot 2^x}{3} = \frac{500}{3}$$

$$2^x = \frac{500}{3}$$

$$\lg 2^x = \lg \left(\frac{500}{3} \right)$$

$$\frac{x \cdot \lg 2}{\lg 2} = \frac{\lg \left(\frac{500}{3} \right)}{\lg 2}$$

$$x = \frac{\lg \left(\frac{500}{3} \right)}{\lg 2}$$

$$x \approx 7,38$$

(31)

$$y = C \cdot x^a$$

$$y = 100\% - 5,2\% = 94,8\% \text{ ger förändringsfaktorn } 0,948$$

$$C = 1$$

$$x = ?$$

$$a = 2011 - 1991 = 21 \text{ år}$$

$$\Rightarrow 0,948 = x^{21}$$

$$x = 0,948^{\frac{1}{21}}$$

$$x = 0,25\%$$

(32) se facit

$$\left. \begin{array}{l} \frac{1}{a} + \frac{1}{b} = \frac{1}{f} \\ a = \\ b = 600 \text{ mm} \\ f = 50 \text{ mm} \end{array} \right\} \Rightarrow \frac{1}{a} + \frac{1}{600} = \frac{1}{50} \quad \text{Multiplikera med } 30000$$

$$\frac{30000}{a} + \frac{300000 \cdot a}{600} = \frac{300000}{50}$$

$$\begin{array}{r} 30000 + 500a = 600a \\ - 500a \quad - 500a \end{array}$$

$$\frac{30000}{550} = \frac{550a}{550}$$

$$a = 54,54 \dots$$

$$a = 55 \text{ mm}$$

$$b) \quad \frac{1}{a} + \frac{1}{b} = \frac{1 \cdot b}{a \cdot b} + \frac{1 \cdot a}{b \cdot a} = \frac{b}{a \cdot b} + \frac{a}{a \cdot b} = \frac{a+b}{ab}$$

34) Se fact

35) Se fact

36) $6x^2 + x - 1 = 0$
 $x^2 + \frac{1}{6}x - \frac{1}{6} = 0$

Hitte nullstellen

$$x = -\frac{1}{12} \pm \sqrt{\left(\frac{1}{12}\right)^2 + \frac{1}{6} \cdot \frac{24}{24}}$$

$$x = -\frac{1}{12} \pm \sqrt{\frac{1}{144} + \frac{24}{144}}$$

$$x = -\frac{1}{12} \pm \sqrt{\frac{25}{144}}$$

$$x = -\frac{1}{12} \pm \frac{5}{12}$$

$$x_1 = \frac{4}{12} = \frac{1}{3}$$

$$x_2 = -\frac{6}{12} = -\frac{1}{2}$$

$$6 \left(x - \frac{1}{2}\right) \left(x + \frac{1}{3}\right)$$

Factorform

$$2 \cdot 3 \left(x - \frac{1}{2}\right) \left(x - \frac{1}{3}\right)$$

$$2 \left(x - \frac{1}{2}\right) \cdot 3 \left(x - \frac{1}{3}\right)$$

$$(2x - 1)(3x - 1)$$

$a < c$ offenbar $2 < 3$

$b > d$ offenbar $-1 > 1$

$$(37) a) x_1 = -2 \quad x_2 = 1 \quad x_3 = 2$$

$$b) y = k(x+2)(x-1)(x-2)$$

Låt $x=0$ och $y=4$ (avläst i grafen)

$$4 = k(0+2)(0-1)(0-2)$$

$$4 = k \cdot (2)(-1)(-2)$$

$$4 = k \cdot 4$$

$$k = 1$$

$$\text{det ger } y = (x+2)(x-1)(x-2)$$

(38)

$$u(x) = E(x)$$

$$40(15-x)^2 = 100x - 840$$

$$40(225 - 30x + x^2) = 100x - 840$$

$$\begin{array}{r} 9000 - 1200x + 40x^2 = 100x - 840 \\ -840 \quad -100x \quad -100x \quad +840 \end{array}$$

$$40x^2 - 1300x + 9840 = 0$$

$$x^2 - 32,5x + 246 = 0$$

$$x = 16,25 \pm \sqrt{(16,25)^2 - 246}$$

$$x = 16,25 \pm 4,25$$

$$(x_1 = 20,5) \quad x_2 = 12$$

ligger utomför

definitionsområdet

Pq-formeln ger

svår: 12 kr/kg

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$$y = C \cdot x^a$$

$$y = \frac{C}{2}$$

$$C = C$$

$$a = 10$$

\Rightarrow

$$\frac{C}{2} = C \cdot x^{10}$$

$$0,5 = x^{10}$$

$$x = 0,5^{\left(\frac{1}{10}\right)}$$

$$x = 0,93$$

$$y = C \cdot 0,93^x$$

$$y = C \cdot 0,93^5$$

$$y = C \cdot 0,71$$

svare: 71%

40

$$f(x) = \frac{1}{x}$$

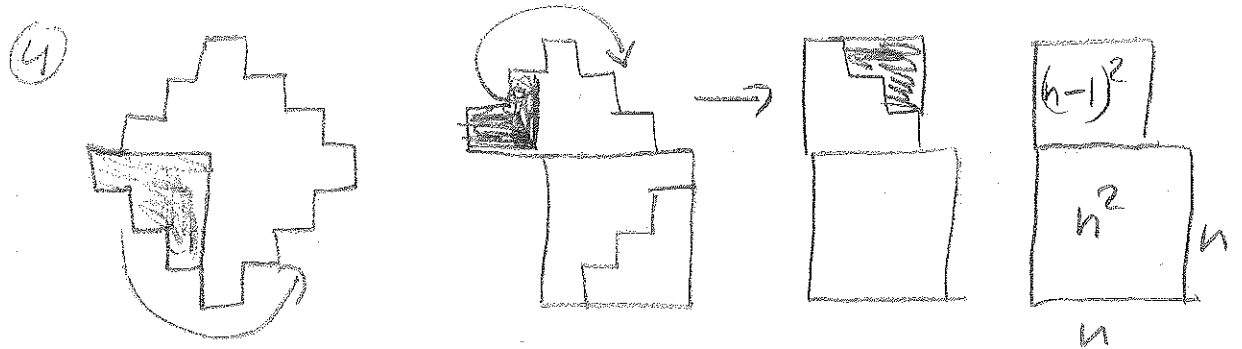
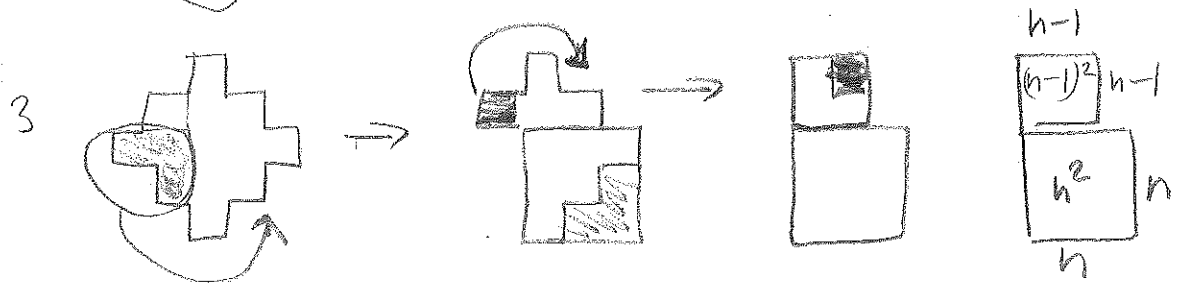
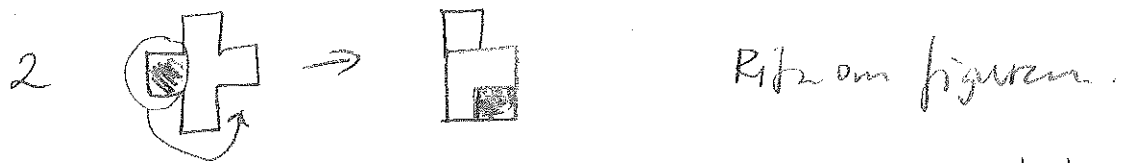
$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x \cdot \frac{1}{x(x+h)} - \frac{(x+h) \cdot 1}{(x+h)x}}{h} =$$

$$= \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} = \frac{x - x + h}{x(x+h)h} =$$

← OBS! Teckenbyte

$$= \frac{-h}{x(x+h)h} = \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{h} = -\frac{1}{x(x+h)}$$

41



SVur: $n^2 + (n-1)^2$

42

Für part.

Punkt 3.

$$\frac{x+1}{x+2} - \frac{x}{x+1} = \frac{(x+1) \cdot \frac{(x+1)}{(x+2)} - \frac{x \cdot (x+2)}{(x+1)(x+2)}}{(x+1)(x+2)} =$$

$$= \frac{(x+1)(x+1) - x(x+2)}{(x+1)(x+2)} = \frac{x^2 + 2x + 1 - x^2 - 2x}{(x+1)(x+2)} =$$

$$= \frac{1}{(x+1)(x+2)} \quad \text{VSB}$$

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Se part